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THE ROLE OF GRAPHICAL CALCULATOR IN DEVELOPING MATHEMATICAL ARGUMENTATION[1]

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This paper discusses the role graphic calculators can play in the development of mathematical argumentation in the context of mathematics classroom, either in small or big groups. Acting as a researcher, the teacher designed a research task on rational functions and monitored its application in a 11th grade class. The proposed task was first discussed in small group and later with the whole class. Finally each student prepared an individual report. A qualitative and descriptive research methodology was adopted. Along this experiment the role of graphic calculators was crucial, helping students to understand the proposed task as well as in validating or rejecting previous conjectures, constructing and visualizing graphs of different functions, formulating proof attempts, and therefore contributing to the development of mathematical argumentation.
Keywords: Mathematical argumentation, graphic calculator, research task.

INTRODUCTION

The development of student's argumentation abilities became along the last two decades a major research issue (Reid & Knipping, 2010). This has been driven by the quest on precocious proof skills, the role of social interactions in the development of mathematical competences and the increased relevance of argumentation in curricula as a reinforcement stimulus for students' intellectual autonomy (Douek & Pichat, 2003; Reid & Knipping, 2010).

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Pedemonte (2007) stresses the relevance of argumentation as well as the need for building contexts in the classroom able to encourage experiences focused on mathematical explanation, reasoning and conjecture and proof assessment.

One is forced to recognize, however, that, despite of the increased relevance given to argumentation in mathematics education research, there is still a long way to be made before its role is fully understood and its potential made effective. Consider, for example, the curricular area devoted to functions. Domingos (2008) has an extensive account on the little relevance given to functions and function-based reasoning in Portugal, despite of the theme being part of all curricula at basic and secondary levels. For Kaldrimidou and Ikonomou (1998) the notion of a function has a central role in Science as well in suitable mathematical education. Several studies point out its argumentative potential, arising namely from the interaction between different representations (numerical, graphical, tabular, etc) of functions. And point out as well how difficult it seems to be, for students, even to relate such different representations (Carraher & Schliemann, 2007; Leinhardt, Zaslavsky & Stein, 1990). Actually, graphical representation of functions, being a major topic in mathematical education, is often regarded by students as a wall difficult to climb up. It seems that, working with functions and graphs, their knowledge becomes fragmented and unable to make to figure out the right connections between them (Demana & Waits, 1992).

Graphic calculators bring a whole new dimension to this scenario, mainly through the possibility they offer to visualize the graph of a function while reasoning about (Demana & Waits, 1992). Visualization is the mental process of joining together several images around a specific concept or process, aiming at a clear identification of the concept/result in discussion. Depending on the context of its introduction, graphical rendering of functions is not disconnected from other forms of representing and reasoning about functions (Cunninghan & Zimmermann, 1991). The possibilities offered by a graphical calculator allow students to devote more time to problem solving and strategic reasoning, and less so the basic mechanic manipulation of data (Kaber & Longhart, 1995).

But what can be achieved by a student equipped nowadays with a graphical calculator seems most promising. First of all this sort of devices encourage discussion in the classroom, with students sharing conjectures, tests and proof attempts in an atmosphere close to that of mathematics' research (Gracias & Borba, 2000). Easy manipulation of graphical representations and tools offers several opportunities for training student's argumentation skills, see e.g. Dugdale (1993).

To put into practice the research experience reported in this paper, the teacher-researcher selected and implemented a specific task, on rational functions, in a class of the 11th grade. The task is part of a sequence aiming at exploring functions and related concepts. Its purpose is not only to provide a meaningful experience on the notion of rational function, but also to develop students' argumentation skills.

In such a context, the paper addresses the following question: May a graphical calculator serve as a mediator and facilitator in the process of developing suitable mathematical argumentation skills, whenever students are challenged to explore a specific task either in small or big group?

BACKGROUND

Mathematical argumentation

Traditionally the focus of school mathematics is the product (concept, result) of the learning process, and not the process itself (i.e., the way from which a conclusion is reached). This maybe at least partially explained by the fact that students have often a lot of difficulties in explaining or justifying their own reasoning (Vincent, Chick & McCrae, 2005). Making mathematics, however, consists of performing several activities involving discoveries, conjectures, generalizations, counterexamples, refutations and proofs (Reid & Knipping, 2010). Errors are common along this process but even them regarded as learning opportunities.

When in the classroom a student explains the way she has thought about a specific problem, her argument makes a difference in what follows. Actually, several fellow students will react by developing further arguments or refutations (Whitenack & Yackel 2008).

Modern curricular guidelines in Mathematics emphasize the need for special contexts, in the class, in which explanations and justification of ideas or mathematical play a major role (Boavida, 2008). This entails the need for dynamics able to involve all students in argumentative activities, at all stages of development and in every item of the curriculum.

Attitudes which could be stimulated include: always regard statements as conjectures, so that the ability to test and change one's own statements until reaching convincing justifications (Reid & Knipping, 2010); test conjectures and explain them (Whitenack & Yackel 2008); and keep a critical regard towards arguments proposed by others.

The Graphic Calculator

Graphic calculators were not introduced in the classroom practice as a mere calculational tool to support the usual activities. On the contrary, its purpose was to enable students to solve more complex, challenging problems which would almost impossible to tackle otherwise in a classroom context. To make this possible, however, and to fully explore all the potential of such a tool to develop students' autonomy and critical sense, the use of graphical calculators should be monitored carefully (Quesada, 1996). Note that several authors underline the fact that most students resort to a graphic calculator as handy tool to confirm previous results obtained by analytical means (Rocha, 2000).

An informed use of graphic calculators requires from the teacher an extra responsibility in proposing and planning relevant tasks. Similarly, each student has to assess whether they constitute the right tool to use in face of each problem proposed (Burril et al., 2002). In a broad sense, it is consensual that graphic calculators brought to classroom a whole new sort of tasks, questions and strategies for the teaching/learning process (Dunham & Dick, 1994).

Rocha (2000) argue that using the graphic calculator in addressing challenging tasks provides an additional stimulus for students to elaborate, analyse and discuss their conjectures, promoting research skills and the ability to formulate logical arguments. The calculator opens new possibilities for research projects carried on in the classroom, as it offers students an expedite way to formulate new conjectures and counter-examples, as discussed for example in (Hirschhorn & Thompson, 1996). Summing up, there seems to be already strong evidence in the literature on the role graphic calculators may have in helping students to become active agents of their own learning processes.

METHODOLOGY

In order to characterize the role graphic calculators may play in the development of argumentation skills in the Mathematics classroom, a concrete experiment was planned and implemented along February 2010 in a 11^{th} grade class. This was an heterogeneous class, with 25 students (18 female, 7 male), in which the researcher, identified as this paper's first author, was simultaneously the Maths teacher.

The experiment consisted of a sequence of tasks through which students were supposed to developed their own understanding of the concept and behaviour of a rational function. In the sequel exposition is limited to extracts of the fifth, and last, task in the sequence.

Data was gathered through audio and video recording of student's dialogues either in small groups or involving the whole class. Later this was complemented by student's individual reports and personal reflection on the proposed task.

Data analysis was split in two categories: mathematical argumentation (formulation and test of conjectures and how to go from conjecture to proof) and graphic calculators (contributions and difficulties). In this paper attention is limited to a number of small excerpts of dialogues in small group, as well as research reports and reflection on the task.

THE RESEARCH TASK EXPERIMENT

To carry on this experiment, the class was divided into several small groups, with three or four elements each. The proposed task consisted of analysing two graphs representing the same function, obtained from different visualization windows in a graphical calculator (see Figure 1). The challenge was to identify the corresponding analytical expression (1).

$$f(x) = ax + b + \frac{c}{dx + e} \quad (1)$$

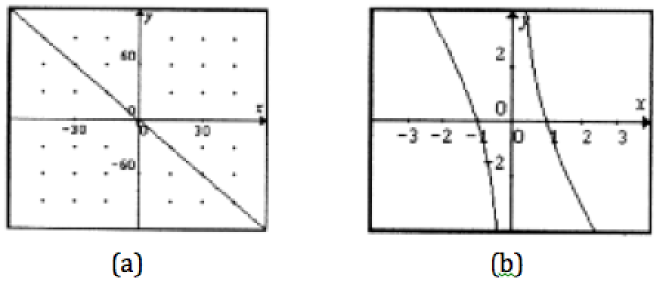

Figure 1. Different visualization windows in a graphical calculator

The task's aimed at calling students attention to what is involved when choosing a specific visualization window in the graphic calculator to obtain the graph of a specific function. This was discussed in small groups resorting to graphic calculators. Later, in the class discussion, an interactive whiteboard to which the calculator was previously connected.

Task implementation within the small group

As a starting point the teacher made a brief introduction recalling that the family of functions to be studied has the following general form (1). She added that the graphical representation of this family of functions could have oblique asymptotes, a situation not found in all other cases previously studied in this class.

Mathematical argumentation

In an initial phase, students start analysing two graphical representations of the same function, for which one wanted to compare the respective analytical expression. Most of the groups tried to identify the equation of the line depicted in Figure 1(a). One of groups immediately suggested

$$y = -2x \quad (2)$$

and discussed its validity as follows:

Fausto: According to the text both graphs represent the same function.

Elisa: Indeed. What changes is the scale. We have to find out the expression.

Fausto: The expression must be like $f(x) = ax + b + c / (dx + e)$.

Elisa: It seems the line in Fig. 1(a) corresponds to the asymptote in Fig. 1(b).

Alexandra: Let's compute the line expression, then.

Fausto: The expression is $y = -2x$.

Júlia: Now we have to connect the expression with something …

Elisa: As the expression of the oblique asymptote, which is $y = ax + b$, let's compare.

Alexandra: We already know: $ax + b$. This $ax + b$ is equal to $-2x$.

This group had no difficulty in connecting both graphs given because is arrived easily to a conjecture relating the equation of the line in Figure 1(a) with the oblique asymptote in Figure 1(b). Other students, tried first to find an equation for line in Figure 1(a). The equation was determined from two points in the graph (30,-60) and (-30,60).

Célia: … from Fig. 1we may conclude that … Shall we determine the line pending?

Raul: Yes.

Célia: Let's take point in the line segment *A (30,-60).*

Raul: We may also consider point *B (-30,60).*

Margarida: Let's compute the line pending with those two points.

Raul: It's *AB = B-A = (-60, 120).*

Célia: The pending is then *120 / (-60),* which is *-2*. Take the line equation *y = mx + b*

… We replace *y* by *-60*, *m* by *-2* and *x* by *30*. Then we compute the *b* and find out *b=0.*

Thus, the equation is *y = -2*.

Afterwards, all groups tried to find out the analytical expression for the function sketched in Figure 1. At this stage students proposed several conjectures and tried to verify them in a way independent of the visualization window used. One of the groups had this discussion about the most relevant aspects of Figure 1(b).

Célia: From Fig. 2 we may take points *C (-1,0)* and *D (1,0).* The vertical asymptote is

x = -d/c.

Raul: The vertical asymptote is *x = 0*.

Célia: Let's write the analytical expression of a function whose asymptote is x=0. So,

f*(x) = ax + b + c /(dx + e)* …

Raul: If the vertical asymptote is zero, *e* must be zero as well.

Célia: Therefore, *f(x) = ax + b + c /dx*. Moreover, *ax + b* becomes *-2 x* in the function.

Students in this group later discovered, by trial and error, the real analytical expression. The graphic calculator was central for them to reach such a conclusion. However, they also had the need to verify its validity by classical mathematical argumentation.

Raul: By trial and error I discovered that the function is *f(x) = -2x + 2/x*

Margarida: We have to do it analytically now …

Célia: We know function is *f(x) = -2x + c/dx,* as *e* is zero.

Raul: Let us replace x and y by one the function zeroes.

Célia: Replacing in the function point *(1,0)* entails *c = 2d*.

Margarida: So, if we replace *d* by *1* and *c* by *2* …

Raul: … we get function *f(x) = -2x + 2/x.*

The majority of groups, in order to test the validity of the analytical expression, found out, and later confirmed, that the value of unknown *c* was equal to the double of *d*. Later they also concluded that this has to be indeed the case and furthermore both unknowns must have the same signal.

Alexandra: The value of *c* doubles *d*.

Fausto: Yes, of course. Then, if *d* is *1*, *c* must be necessarily *2*.

Júlia: Let's check it out, ok? Replace *d* by *2* and *c* by *4*.

Elisa: The value of *c* must double that of *d*, but their signal must be equal. If they are different we fail again.

Alexandra: Yes, in such a case expression *–d/e* does not apply.

Fausto: Then the expression is $f(x) = -2x + c/dx$.

Other groups were unable to reach the conclusion that *d* doubles *c*. However, by trial and error they were able to find the relevant analytical expression.

Rui: Now we have to think of values for *c* and *d* …

Maria: … until we get the graph in Fig. 1 … *c=1* and *d =-1*.

Rui: No way.

Dora: *c=1* and *d =-1*.

Vitória: Not quite, but we are approaching. Try *c=2* and *d=1*.

Rui: Perfect! It's done!

By the end of this task all students reported explaining the way they reasoned to accomplish it. In their final reports students mentioned they started with a analysis if each given graph. Most of the reports show their ability to organize reasoning. They always use a complete and rigorous argumentation concerning conjectures made or discarded, and proof attempts.

The final part of students' reports contains a personal reflection on the whole experiment. Célia, for example, mentions the task "had a very important role in my learning process and, in particular, my ability to argue with respect to conjectures defended or abandoned." She also underlines that this sort of active learning is "great and increases our willingness to go the school".

The graphic calculator

The graphic calculator was used to support students' arguments and activities in this task, in particular to test whether the analytical expressions proposed matched the graphical representations in Figure 1, through a suitable change in the visualization window. From the data gathered in all groups its relevance to this purpose is obvious. The use made of this tool was always active and critical. For example, the following excerpt shows how a group resorted to the *Zoom* facility.

Dora: To be sure it's ok, better to increase the window.

Maria: Why?

Rui: The bigger the window, closer the function gets to its asymptote.

All: Yes!

Dora: Thus the analytical expression for both cases is $f(x) = -2x + 2/2$.

Other groups did interesting experiments forcing parameters to change and anticipating, and later confirming, results. Students' reports provide evidence on the major role played by the graphical calculator in this task. Raul, for example, reports how by modifying the visualization window it was possible to conclude that both graphical representations referred to the same function (Figure 2).

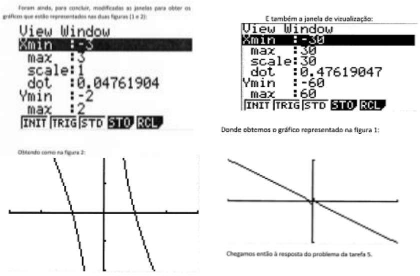

Figure 2. Excerpt of Raul's report

Sónia, on the other hand, mentioned how she resorted to the menu table to confirm that function had a vertical asymptote (see Figure 3).

x	Y1
-2	3,8333
-1	1,6666
0	ERROR
1	-1,6666
2	-3,8333

Figure 3. Excerpt of Sónia's report

Another student, Célia, commented on the value of a graphical calculator to "validate or refute a conjecture". She warned, however, for the need of developing the right skills to work with graphical calculators so that "their technical limitations don't lead to wrong conclusions". A similar opinion is shared by Afonso who argues: "[the graphical calculator] is a most useful tool when correctly used, with the most suitable visualization window". Rafaela focused on the relevance of technology for the maths class, as a "support to debate [mathematical] ideas". She writes "new technologies are good companions for Mathematics".

CONCLUSIONS

By exploring graphical calculators, students acquired, along this experiment and progressively, a more critical and reflexive attitude with respect to the proposed task. In particular, they made progress when trying to identify regularities among possible solutions or partial solutions A similar conclusion is presented in Gracias and Borba (2000) who defend that graphical calculators induce new learning opportunities based on experimentation and group discussion.

Building and visualizing graphs of different functions was a major advantage referred by several students. The role of visualization in the complex process of knowledge building (see, e.g. Demana and Waits (1992). In summary, the relevance of graphical calculators for the successful accomplishment of this task is clearly supported by the empirical data collected.

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